Chapter One

Introduction to Quarter-Tone Music

There is little mention of microtonal music in primary academic scholarship, and what little exists often comprises only a brief *en passant* reference to microtonal music embedded within a topic that deals primarily with non-microtonal music. Even though the repertoire of microtonal compositions is growing, theorists have not yet isolated some of the basic issues that this music engages. In this dissertation, I begin to address this lacuna by examining quarter-tone music.[1] This introductory chapter considers fundamentals of quarter-tone music theory such as pitch names, enharmonic equivalence, intervals, chords, and scales, as well as proposing notational conventions and terminology for discussing quarter-tone music. Chapters 2 through 5 present analyses of selected quarter-tone works by Easley Blackwood, Alois Hába, Charles Ives, and Ivan Wyschnegradsky.

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[1] Quarter-tone music represents one of an infinite number of possible microtonal systems. We can divide microtonal systems into one of two categories: (1) equal-tempered systems, in which the octave (or some other interval, such as the perfect twelfth) is divided into an arbitrary number of equal intervals, or (2) intervallic systems, in which specific interval ratios (such as the just intervals of the harmonic series) generate the pitch content of the microtonal space. Quarter-tone music is normally equal-tempered, dividing the octave into twenty-four equal intervals. Restricting this study to a single tuning system allows me to discuss both the quarter-tone repertoire in detail, and specific implications that pertain to a broader range of microtonal systems.

The four composers featured in these chapters are ordered in a progression leading from Blackwood, the most conservative quarter-tone composer, to Wyschnegradsky, the most progressive. Chapter 6 extends Richard Cohn's "parsimonious trichord" by applying the transformational approach of neo-Riemannian theory to chords derived from a quarter-tone scale developed by Wyschnegradsky.[2]

One obstacle to overcome when analyzing microtonal music is that most of our analytic tools depend on the assumption that the octave is divided into exactly twelve equal parts. From our earliest days of musical training, everything we learn about basic musical concepts (such as semitones, note names, intervals, and enharmonic equivalence) is built upon this assumption. When analyzing music for which this assumption does not hold, our analytical tools often can seem inadequate. In order to analyze microtonal music, then, we need to rethink basic assumptions about pitch, starting with the rudiments of music, before progressing to more complex analytical observations. Here, I begin with the basics of music notation by proposing notational conventions that I use throughout the dissertation.

[2] Richard Cohn, "Neo-Riemannian Operations, Parsimonious Trichords, and Their *Tonnetz* Representations," *Journal of Music Theory* 41/1 (1997).

If we divide pitch space into equal-tempered quarter tones, we derive twenty-four pitches in each octave. Most composers use quarter tones as though they were extra pitches inserted into the middle of each of the familiar semitones, and therefore in many cases it is useful to think of the full set of quarter tones as partitioned into two sets: one set containing the twelve familiar pitches, and the second containing the new quarter-tone pitches. Throughout this dissertation, I use the term "conventional" to refer to familiar musical entities. Conventional musical objects are those that would be familiar to any trained musician—examples of such objects include conventional pitches (such as F♯ or E♮), conventional intervals (such as the minor third or the tritone), or conventional chords (such as the major triad or the dominant seventh chord). I use the term "quarter-tone" to refer to the new musical objects that are created when we introduce quarter tones into conventional twelve-note equal-tempered pitch space. One difficulty in interpreting quarter-tone music is that composers notate quarter tones in different ways. For each quarter-tone composer, we likely will need to master a new notation for quarter tones, an obstacle that makes it difficult to compare quarter-tone practices among different composers. At an early stage in my own quarter-tone studies, it became obvious that I would need to develop and apply consistently my own notational system. My initial premise was that new symbols should be easy for a traditionally-trained musician to learn, remember, and understand. Thus, I first need to defend my chosen conventions for naming quarter-tone pitches, pitch classes, intervals, chords, and sets.

Quarter-Tone Notation

Quarter-tone composers typically use the conventional norms of music notation as a point of departure for their own notation. Gardner Read, who has surveyed the music of hundreds of microtonal composers, identifies four common methods for notating microtones: (1) small ancillary signs placed near the noteheads of microtonal pitches; (2) shaped noteheads that differentiate microtones from conventional pitches; (3) numeric representations of microtones; and (4) new accidental signs that add microtonal accidentals alongside the conventional sharps, flats, and naturals.[3] One additional method should be added to Read's list: it is common, especially in keyboard music, to use conventional notation

[3] Gardner Read, *Twentieth-Century Microtonal Notation* (New York: Greenwood Press, 1990), 10.

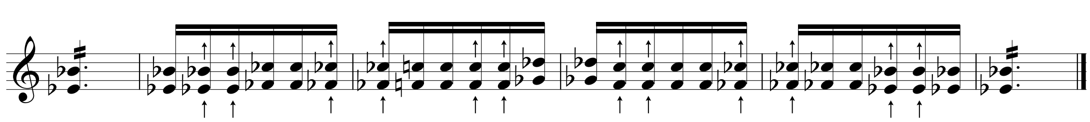
exclusively, but to then include instructions for playing on an instrument tuned either one quarter tone sharp or one quarter tone flat.

Example 1.1: Charles Ives, *Three Quarter-Tone Pieces,* Mvt. II, *Allegro,* mm. 3-6

In *Three Quarter-Tone Pieces,* Charles Ives notates the score as if for piano duet in which both grand staves are notated conventionally along with instructions that the *piano primo* should be tuned one quarter tone higher than normal, while the *piano secondo* should be left at regular pitch. In this scoring, one keyboard plays the twelve conventional pitches, and the other plays the twelve quarter-tone pitches; thus, between the two keyboards, all twenty-four pitches are produced. Other composers have used a similar scheme: John Eaton's *Microtonal Fantasy: For One Player at Two Pianos* is scored as though for piano four-hands with the *piano secondo* tuned down one quarter tone; Klaas de Vries tunes two harpsichords in a similar manner in his incidental music composed for a play based on Canadian author Margaret Atwood's short story *Murder in the Dark.*

From the pianists' perspective, scoring for piano duet is easy to perform since the notation is not unconventional—the pianists simply play what is written and the tuning of the pianos produces the quarter tones. The piano with an altered tuning is a transposing instrument. Even though scoring for piano duet is convenient for performers, a short excerpt from Ives's *Three Quarter-Tone Pieces* (see Example 1.1) demonstrates how it can challenge analysts. In m. 3, the *primo* presents a grace-note motive that descends chromatically, leading to the pitch E♮; this motive is then echoed by the *secondo.* The *primo* repeats the motive, transposed down a semitone, and is again echoed by the *secondo.* This results in the motive moving through a sequence based on successive transpostions descending by quarter tone, but this descending sequence is concealed by the score. The staccato eighth-notes in mm. 4-5 likewise form a line of descending quarter tones, but it is difficult to see the linear connections because the line jumps from one keyboard to the other. In cases such as these, it can be difficult to recognize which pitches are altered and which are not, especially in complex textures where melodic lines are exchanged between *primo* and *secondo.* Scoring for piano duet can also conceal the structure of harmonies when chords are comprised of a combination of conventional and quarter-tone pitches. In Example 1.1, the structures of the complex eight-note chords in m. 4 are relatively easy to see since the pitches of the two pianos are kept registrally separate, but the structures of the chords in m. 6 are more difficult to determine by inspection because the two keyboards overlap, thereby mixing pitches.

Example 1.2: Ernest Bloch, *Quintet for Piano and Strings,* Mvt. I, *Agitato,* mm. 1-3



Example 1.3: Béla Bartók, *Sonata for Solo Violin,* Mvt. IV, *Presto,* mm. 206-211

Some composers use conventional notation, but then add small signs to indicate quarter-tone inflections. In the opening section of the first movement of Ernest Bloch's *Quintet for Piano and Strings* (Example 1.2), the small slash indicates that the following pitch, in this case C♮, should be raised one quarter tone. A different convention appears in Bartók's *Sonata for Solo Violin* (Example 1.3), where he uses a small upward-pointing arrow to raise pitches by one quarter tone. In general, ancillary signs such as these are typically used by composers who are using quarter tones as auxiliary microtones that embellish an otherwise conventionally-tuned texture.

Example 1.4: Excerpt from Schubert, *Symphony No. 8:* a) conventional noteheads; b) square-shaped noteheads

Example 1.5: Jack Behrens, *Quarter-Tone Quartet,* op. 20, mm. 1-2

A different approach for notating quarter tones uses regular noteheads to indicate conventional pitches and specially-shaped noteheads to indicate quarter-tone pitches.[4] I have illustrated one possible system of shaped noteheads in Example 1.4. Example 1.4a is notated with conventional noteheads, and therefore sounds at written pitch; Example 1.4b, notated with square noteheads, sounds one quarter tone higher than written. In his *Quarter-Tone Quartet,* Jack Behrens uses special noteheads in combination with special tuning instructions (Example 1.5). In the score, Behrens indicates that the first violin and viola are to be tuned one quarter tone higher than usual, leaving the second violin and cello at regular pitch.[5] Behrens also indicates

[4] Read, 62-63. Many different shapes have been employed for the purpose of notating quarter tones, including squares, triangles, diamonds, and crosses.
[5] Behrens's scoring obscures the fact that in Example 1.5 the interval between the pizzicato eighth-notes in the viola (which sound one quarter tone higher than written) and the sixteenth notes sounding simultaneously in the cello is always a perfect octave.

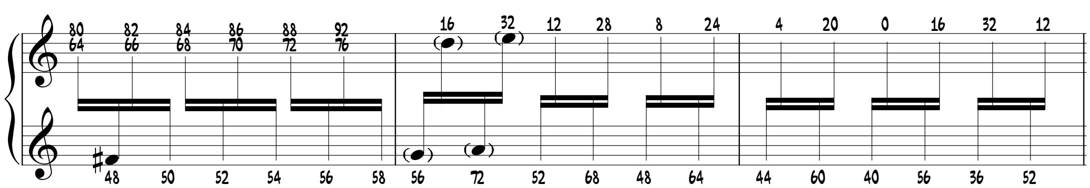
that “notes with the stem on the ‘wrong’ side of the notehead are to be played a quarter tone higher.”[6]

Example 1.6: J. S. Bach, *Invention No. 13,* mm. 1-2: a) conventional notation; b) Carrillo’s notation

Julián Carrillo’s microtonal notation blends conventional notation with pitch-class notation. Carrillo replaces noteheads with pitch-class integers and the conventional five-line staff with a single line that represents middle C. The pitch C♮ is always represented by zero. The pitches from C_4 through B_4 are notated on the single staff-line; the pitches from C_5 to B_5 are notated above the line; the pitches from C_3 to B_3 are noted below the line. More remote octaves lying outside this range require leger lines. Example 1.6

[6] Jack Behrens, *Quarter-Tone Quartet,* op. 20 (Toronto: Canadian Music Centre, 1972).

shows the first two measures of Bach's *Invention No. 13* (BWV 784) in conventional notation on the top two staves (Example 1.6a), and renotated in Carrillo's notation on the bottom two staves (Example 1.6b). The A_2 in the left hand requires a leger line because it falls below C_3. Example 1.6 requires integers from 0 to 11; quarter-tone music, with twenty-four pitches in each octave, would require integers from 0 to 23. Carrillo's sixteenth-tone music requires integers from 0 to 95 to notate pitch-classes in an octave divided into ninety-six equal parts.

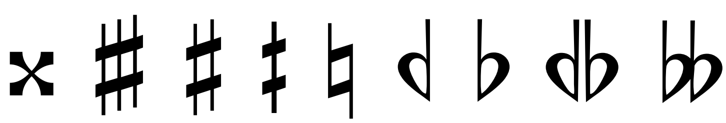


Example 1.7: Julián Carrillo, *Horizontes,* harp part, 5 measures after rehearsal letter E *(Misterioso)*

Carrillo believed that his notation would make music more accessible to readers with no musical training; in his view, the use of integers in place of traditional notes would make reading music "as easy as reading the newspaper."[7] However, as we can see from the short excerpt from the harp

[7] Stanley Sadie and John Tyrell, eds. *The New Grove Dictionary of Music and Musicians,* 2d ed. (New York: Macmillan Publishers Limited, 2001), s.v. "Carrillo, Julián," by Gerald R. Benjamin.

part (tuned in sixteenth tones) taken from *Horizontes* (Example 1.7), Carrillo's notation is not easy to read, and in fact it presents some serious obstacles for performers.[8] While the integers in the first measure of Example 1.7 follow a simple sequential pattern (moving up by an interval of two sixteenth tones each time), the integers in the following measures progress in a more complex pattern that would be difficult to realize musically.[9] While Carrillo's notation does create problems for performers, the integer notation is convenient for analysts because it is easier to compute interval sizes and to compare pitch-class sets since there is no need to convert pitches into pitch-class integers.



Example 1.8: Quarter-Tone Accidentals

[8] Carrillo's orchestral scores acknowledge the impractical nature of his integer notation. As published by *le Societé des Editions Jobert,* Carrillo's music uses integers only for the harp parts. For the remaining orchestral instruments, he replaces the integers with conventional notation modified by ancillary marks similar to the slashes used in Example 1.2. The noteheads in Example 1.7 appear in the Jobert Edition of the score and help orient the harpist. In 96-tone equal temperament, pitch class 48 is equivalent to F♯, pitch class 56 is equivalent to G♮, and so on.
[9] The integers in mm. 2-3 of Example 1.7 are all evenly divisible by four, which means that the smallest interval in these measures is four sixteenth tones or exactly one quarter tone. It would be possible to renotate these two measures using some form of quarter-tone notation.

Most microtonal composers use some kind of modified accidental signs to notate a quarter-tone pitch. Some composers add arrows to the conventional accidental signs to indicate that a pitch should be raised or lowered by one quarter tone, while others create a new set of quarter-tone accidentals to add to the familiar set of conventional accidentals. Because there is no accepted standard for quarter-tone accidentals, I have chosen a single set of accidentals to use throughout this dissertation. In my musical examples, I use the accidental signs shown in Example 1.8, transcribing each composer's individual notation into my chosen notation.

The reversed flat sign (𝄳) represents one quarter tone flat; the conventional flat sign, two quarter tones or one semitone flat (its usual interpretation). The sign for three quarter tones flat (𝄳♭) combines the reversed flat sign and the conventional flat sign. The conventional natural sign has its regular meaning. The sharp sign with one vertical stroke (𝄲) indicates that the note is raised one quarter tone; the conventional sharp sign indicates that the note is raised two quarter tones or one semitone; and the sharp sign with three vertical strokes (𝄰), that the note is raised by three quarter tones. The double sharp (𝄪) and double flat (𝄫) are also available with their conventional interpretations; however, in the quarter-tone music I have analyzed, double accidentals appear only rarely, if at all.

	Blackwood	Hába	Wyschnegradsky	Skinner
3/4 sharp	↑♯	♯↘	♯𝄲	♯𝄲
1/4 sharp	↑♮	↘	𝄲	𝄲
1/4 flat	↑♭	♭	♭	𝄳
3/4 flat	none	♭♭	♭♭	𝄳♭

Table 1.1: Quarter-tone Accidentals Compared

In Table 1.1, I show the accidentals used by Blackwood, Hába, and Wyschnegradsky along with my preferred accidentals. Blackwood does not create new quarter-tone accidentals, but instead uses an up-arrow (↑) symbol to modify the conventional accidental signs. For this dissertation, I have chosen to use Wyschnegradsky's sharp signs because they work well symbolically; each vertical stroke represents a rise in pitch by one quarter tone, so that the three-stroke sharp sign represents a higher pitch than the conventional two-stroke sharp, which in turn represents a higher pitch than the one-stroke quarter-sharp sign. Hába's three-quarter sharp sign can create visual confusion, especially in hand-written scores, because it can be difficult to distinguish from the conventional two-stroke sharp, and the arrow-like feature on the bottom right corner of the accidental sign is too easily mistaken for an accent mark.[10]

The flat sign lacks any graphical feature that lends itself to simple correspondence between symbol and pitch; thus, the accidentals that I select to represent one- and three-quarter flats may seem arbitrary. I have chosen the reversed flat to represent the one-quarter flat because it is the default accidental sign provided by mainstream notation software packages, such as *Sibelius* and *Finale*.[11] The sign for three-quarters flat logically combines the one- and two-quarter flat symbols; Wyschnegradsky's three-quarter flat sign also follows this logic. Hába's three-quarter flat sign presents graphical problems; the curvature of the flag attached to his three-quarter flat is too similar to the curvature of a notehead and can be visually confusing.

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[10] Many of Hába's scores that are published by Musikedition Nymphenburg are hand-written and can be inconsistent with respect to the size, shape, and placement of notational symbols. In dense textures, Hába's quarter-sharp symbol is easily mistaken for an accent sign, while his quarter-flat symbol often resembles a half-note.

[11] It is always convenient to use the default accidentals available in any given notation software; in fact, it is often difficult to use anything other than the default microtonal accidentals. Because it is a standard glyph in the popular *Opus* and *Petrucci* notation fonts (included with *Sibelius* and *Finale* respectively), the reversed flat is beginning to assert itself as a *de facto* standard sign for representing the one quarter-tone flat accidental.

Basic Musical Objects: Pitch Names, Intervals, and Chords

Establishing a set of quarter-tone accidentals allows us not only to notate pitches on the staff, but also to describe pitches with traditional letter names. To facilitate easy comprehension, I decided that the orthography of the new accidental signs should be identical to that of their conventional counterparts. When writing note names, I place the accidental sign to the right of the letter name of the pitch that it modifies, which is exactly where a trained musician would expect. When speaking the note names aloud, I pronounce “G𝄲” as “G quarter-sharp,” which is short for “G one quarter tone sharp”; I pronounce “G𝄳” as “G quarter-flat.” I read “G♯𝄲” as “G three-quarters-sharp,” and “G𝄳♭” as “G three-quarters-flat.”[12] When writing musical notation, I place the accidental sign immediately to the left of the notehead of the pitch that it modifies. I find that extra care is required in the alignment of quarter-tone accidentals, especially when writing closed-position chords; the ‘𝄲’ sign is narrow and requires extra whitespace in order

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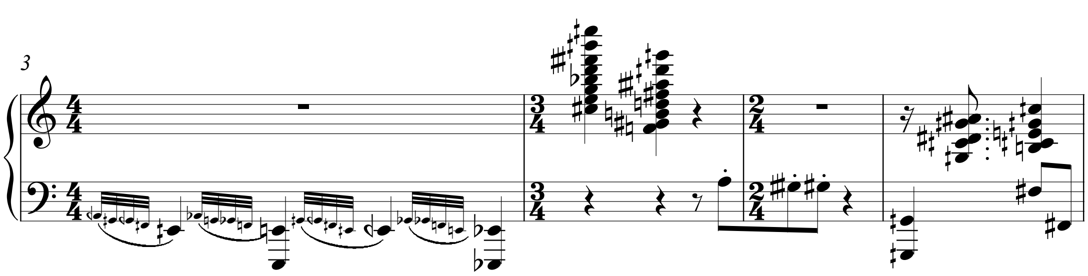
[12] Read prefers the terms “semi-sharp” and “semi-flat” for one quarter-tone accidentals, and the terms “sesqui-sharp” and “sesqui-flat” for three quarter-tone accidentals. I find that it is too easy to confuse the words semi-sharp and semi-flat (indicating a pitch change of one quarter tone) with the word semitone. Moreover, the term “sesqui-sharp” is awkward to pronounce.

to be visible, while the '#' is wider than a conventional sharp and tends to crowd nearby accidentals.

Example 1.9: Alois Hába, *String Quartet No. 3*, mvt. 3, cello part, mm. 7-14, five versions

In Example 1.9, I have written the same melody in five different notations in order to make it easier to compare the relative merits of each. When the melody is arranged on two staves, with the top one sounding one quarter tone higher than the bottom one which sounds at standard pitch (Example 1.9a), it is difficult to see linear connections within the melody because the pitches are distributed between the staves. Example 1.9b is easier to see as a melody, although the use of ancillary arrows privileges the conventional pitches by making the quarter tones look like mere modifications of their conventional counterparts. The shaped notes of Example 1.9c, with square noteheads signifying raised pitches, are hard to read because it is difficult to distinguish between the round and square noteheads. Example 1.9d, which is notated in Carrillo's system, using integers ranging from 0 to 23 to represent the full gamut of quarter-tone pitch classes, gives an idea of how difficult it is to read even a simple melody when notes on the familiar 5-line staff are replaced with pitch-class integers. Even though it takes some time to become familiar with the new accidental signs, I believe that Example 1.9e is the best of the five versions given in Example 1.9. In Example 1.9e, I have written the melody using the accidentals that I use throughout this dissertation.[13]



Example 1.10: Ives, *Allegro,* mm. 3-6 transcribed

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[13] For consistency, I renotate musical examples using my own notation, except when I need to make a point about a composer's original notation.

Example 1.10 transcribes Example 1.1 so that the music that was previously notated on two grand staves now appears on a single grand staff, substituting quarter-tone accidentals for the conventional accidentals that are modified by Ives's tuning instructions. Example 1.10 has two advantages over Example 1.1: (1) it is easier to see the linear connections between the pitches in the descending bass lines E𝄲–E♮–E𝄳–E♭ and A♮–G#–G♯; and (2) it is easier to see the interval structures of the large chords in the upper register. However, there are disadvantages to Example 1.10. As I have presented them, the notes and symbols of Example 1.10 are the same size as the corresponding notes in Example 1.1. The accidentals on the grace notes are difficult to distinguish because of their small size. The quarter-sharp accidentals are hard to read even at the regular size, especially when mixed with conventional sharps and three-quarter sharps; the accidentals on the G𝄲 and A♯ in the right hand of the final measure seem to blend into a single '#' symbol. In general, scores that use quarter-tone accidentals need to be typeset at a larger size than scores that use conventional accidentals exclusively, and they require extra whitespace around the unfamiliar accidentals to improve legibility.

Although most of my musical examples use quarter-tone accidentals, occasionally it is more convenient analytically to use pitch-class integers instead of pitch names. The most obvious way to number the pitch classes would be to use the integers from 0 to 23, so that all operations applied to these integers operate in mod-24 arithmetic. This system, although simple, has a disadvantage because there is already a large, pre-existing body of theoretical literature that uses pitch-class integers to assist in the analysis of conventionally-tuned music. Theorists who work with pitch-class integers quickly become comfortable with the correspondence between integers and the pitches they represent. Those familiar with mod-12 pitch-class integers know that when the pitch C♮ is represented by pitch-class zero (pc 0), C♯/D♭ is pc 1, D♮ is pc 2, and so on. Over time, these integer-pitch correspondences become second nature. Because of my familiarity with mod-12 pitch-class integers, I found mod 24 integers confusing. I know from my experience with conventional pitch-class notation that pc 6 represents F♯. In mod 24, pc 6 instead represents E♭, and F♯ is no longer pc 6, but becomes pc 12. Because it can be difficult to keep track of which integers represent which pitches, I have decided on a system of numbering that minimizes that difficulty. My pitch-class numbers are not integers, but rather decimals. Pitch-classes of the form "n.0" indicate conventional pitches, while pitch-classes of the form "n.5" indicate quarter-tone pitches. Anyone who knows that pc 5 (mod 12) represents the pitch F♮ should have little trouble remembering that in the quarter-tone universe, pc 5.0 represents F♮. Likewise, any reader who knows that F♮ is pc 5.0 and F♯ is pc 6.0 should have little difficulty recognizing that pc 5.5 represents F𝄲, the pitch halfway between F♮ and F♯. In Table 1.2, I show a sample of 24-tone pitch classes alongside possible note names with their enharmonic equivalents. Pitch-class 0.0 is arbitrarily set to C♮.[14]

Pitch Class	Name	Enharmonic Names
0.0	C♮	B♯, D𝄫
0.5	C𝄲	B♯𝄲, D𝄳♭
1.0	C♯	B𝄪, D♭
1.5	C♯𝄲	D𝄳
2.0	C𝄪	D♮, E𝄫

Table 1.2: Pitch-class names and equivalent note names

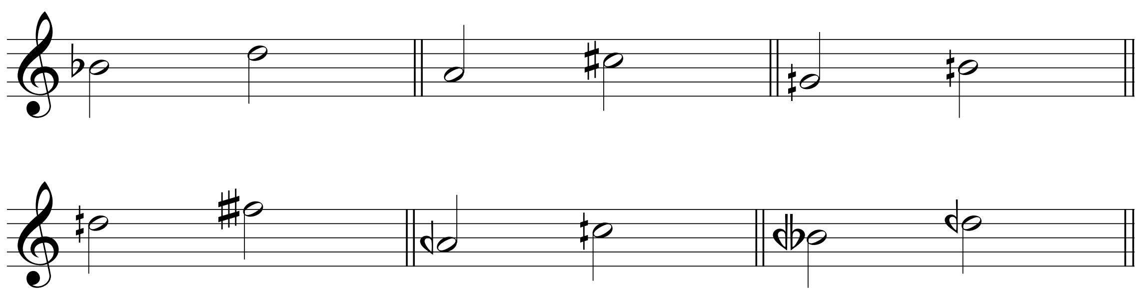
New pitch names and accidentals generate new sets of enharmonically equivalent pitches. Because my system preserves the conventional accidentals along with their usual interpretations, the conventional enharmonic equivalents behave as one would expect. C♯ is enharmonically equivalent to both D♭ and B𝄪, an equivalence relationship that should be familiar. The addition of quarter tones creates new sets of enharmonically equivalent pitches to learn. Throughout the dissertation, I use the

[14] I provide a full table of all twenty-four pitch classes presented with all possible enharmonic equivalents in Appendix A.

mathematical symbol for congruence ("≡") to represent enharmonic equivalence. In other words, "D♯ ≡ E𝄳" is read, "D three-quarters sharp is enharmonically equivalent to E quarter-flat."

In order to discuss the new quarter-tone intervals, we need an appropriate nomenclature. In most cases, I use a numeric measure in place of traditional interval names. As with pitch-class labels, I use decimals for the convenience of readers who identify the conventional intervals as mod-12 integers. If the minor third (or augmented second) can be represented by int 3 (mod 12), then that same interval should be written as the quarter-tone interval int 3.0. Depending on the analytical context, one may prefer to speak of interval class, in which case inversional and octave equivalence is assumed, or simply of intervals, in which case only octave equivalence is assumed. The label "ic 3.0" represents an interval class that is inversionally equivalent to "ic 9.0," while "int 3.0" represents a literal interval spanning three semitones.

In general, I prefer a numeric representation of intervals to descriptive labels. However, whenever I do refer to an interval by name, I do so in such a way that preserves the traditional interval names with their conventional meanings. A whole tone spans two semitones (as one assumes) and each of those semitones itself spans two quarter tones. The general principle is that if an interval looks like a major third, for example, it should still be called a major third. All of the intervals in Example 1.11 are major thirds (int 4.0); the first two are spelled with conventional pitches while the following four use the less-familiar quarter-tone accidentals.[15]

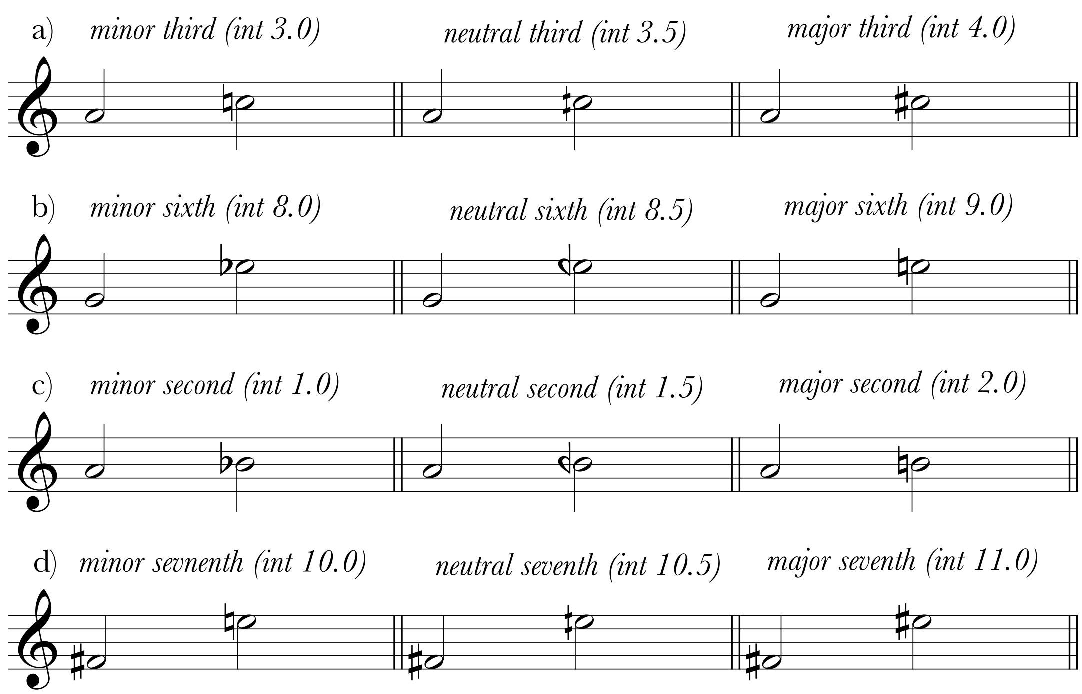


Example 1.11: Six different major thirds

Some writers have identified particular quarter-tone intervals with descriptive names. The most significant of these labels applies to int 3.5, which lies halfway between the minor third (int 3.0) and the major third (int 4.0); Alois Hába called this interval the *neutrale Terz,* or "neutral third."[16] We can locate a neutral interval between each pair of minor and major intervals of equivalent generic size. Therefore, we can identify a neutral third situated halfway between the minor and major thirds as shown in Example 1.12a,

[15] Because the interval between any two pitch-classes of the form n.5 will always be in the form n.0, the interval between any two quarter-tone pitches is always a conventional interval; therefore a major third must be spelled with either two conventional pitches or two quarter-tone pitches, but never one of each.
[16] Alois Hába, *Neue Harmonielehre des Diatonischen, Chromatischen Viertel-, Drittel-, Sechstel-, und Zwoelftel-Tonsystems* (Liepzig: Fr. Kistner & C. F. W. Sigel, 1927), 143.

and we can likewise identify a neutral sixth (Example 1.12b), a neutral second (Example 1.12c), and a neutral seventh (Example 1.12d). Table 1.3 shows that Hába's label for the neutral third is well chosen—not only does the neutral third lie exactly halfway between the equal-tempered minor third and the equal-tempered major third, but it also lies almost exactly halfway between the pure 6:5 minor third and the pure 5:4 major third. Thus, this neutral interval does indeed occupy a type of neutral territory between major and minor.



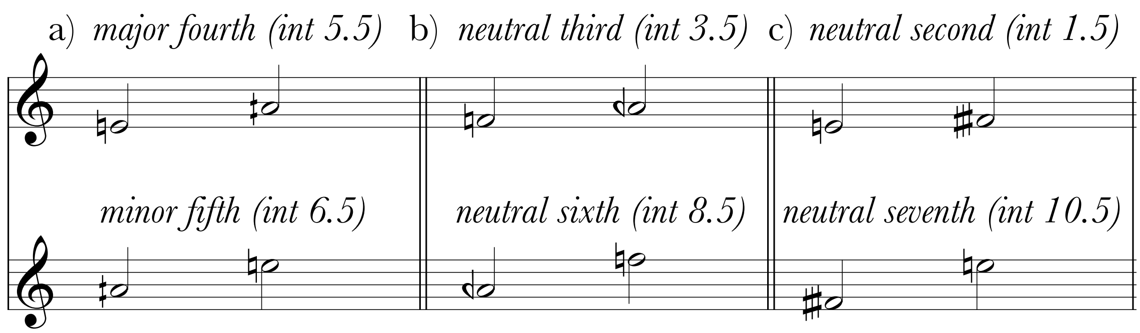
Example 1.12: Derivations of a) neutral third; b) neutral sixth; c) neutral second; d) neutral seventh

Interval	Size (cents)	Difference from 350.0
Equal-Tempered Minor Third	300.0	-50.0
Pure Minor Third (6:5)	315.6	-34.4
Neutral Third	350.0	0.0
Pure Major Third (5:4)	386.3	+36.3
Equal-Tempered Major Third	400.0	+50.0

Table 1.3: Comparison of Thirds

Example 1.13: Wyschnegradsky's major fourth and minor fifth

Ivan Wyschnegradsky applies the label "major fourth" to the interval that lies halfway between the perfect fourth and the augmented fourth (Example 1.13a); he calls the interval between the perfect fifth and diminished fifth a "minor fifth" (Example 1.13b). Wyschnegradsky considers the major fourth (int 5.5) to be an important harmonic interval, because the equal-tempered int 5.5 (550 cents) approximates the ratio of 11:8 (551.28 cents) found in the harmonic series.

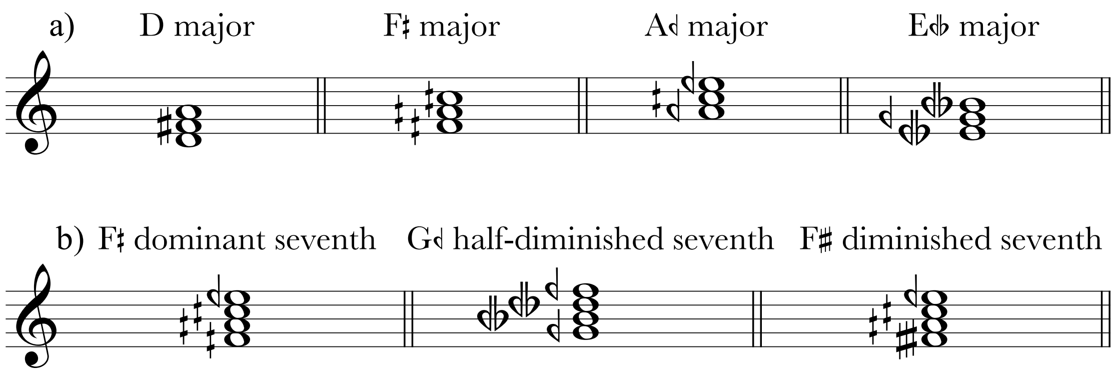


Example 1.14: Quarter-tone intervals and their inversions

One significant feature of both Wyschnegradsky's and Hába's interval names is how they behave under inversion. In conventional music theory, major intervals when inverted always become minor intervals, and fourths when inverted always become fifths. We therefore could predict that Wyschnegradsky's major fourth should become a minor fifth when inverted, and shown by Example 1.14a, this is exactly what happens. Any major fourth (such as the interval E♮-A𝄲 shown in Example 1.14a) always inverts to become a minor fifth (in this case, A𝄲-E♮).[17] Hába's neutral intervals possess an interesting property: the inversion of a neutral interval always produces another neutral interval, just as the inversion of a perfect interval always produces another perfect interval. The neutral third F♮-A𝄳 in Example 1.14b

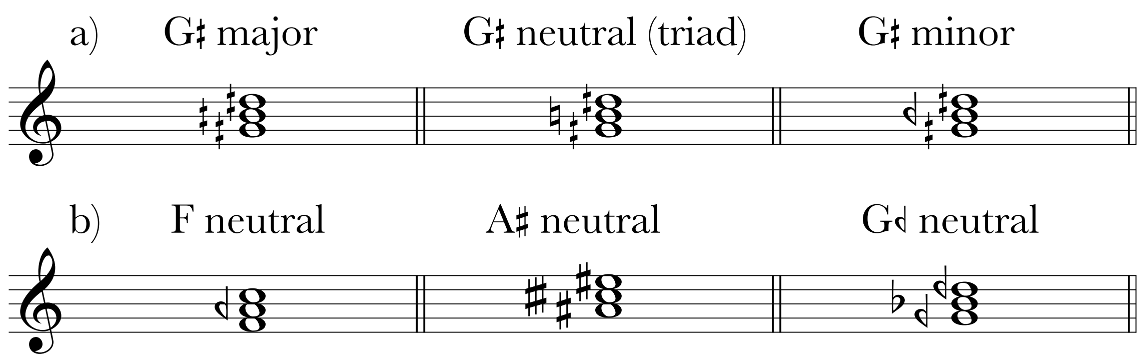
[17] Of course, the reverse is also true: a minor fifth will always become a major fourth whenever inverted. Wyschnegradsky's interval names suggest the theoretical possibility of an inversionally-related "major prime" (int 0.5) and "minor octave" (int 11.5), although I know of no writer who uses these interval names.

becomes the neutral sixth A𝄳-F♮ when inverted, and the neutral second E♮-F♯ in Example 1.14c becomes the neutral seventh F♯-E♮ when inverted.



Example 1.15: Conventional triads and seventh chords

A new set of quarter-tone chords becomes available to quarter-tone composers, in addition to conventional triads and seventh chords. Whenever practical, I use traditional names for conventional triads—a major triad, for example, is composed of a major third and a perfect fifth above its root regardless of whether that root is a conventional pitch or a quarter-tone pitch. The first triad in Example 1.15a is a familiar D–major triad, but the next three triads, F‡–, A𝄳–, and E𝄳♭–major, are unfamiliar, but can be understood as major triads because of their interval structures. A conventional chord is spelled with conventional accidentals if the root is a conventional pitch, and with quarter-tone accidentals if the root is a quarter-tone pitch. The three chords in Example 1.15b are all conventional seventh chords with quarter-tone pitches as roots.[18]



Example 1.16: Neutral triads

Because traditional chord names merely identify conventional chord types, they cannot be used to identify chords containing quarter-tone intervals, although there is one quarter-tone chord that is occasionally referred to as a "neutral triad" that is constructed with a neutral third and perfect fifth above its root. In Example 1.16a, I show a G𝄲–neutral triad positioned between a G𝄲–major triad and a G𝄲–minor triad. The neutral triad, like the conventional diminished and augmented triads, is inversionally symmetrical, since the neutral third (int 3.5) divides the perfect fifth (int 7.0) into two equal

[18] Even after years of study and practice, I find it challenging to identify by inspection conventional chords with quarter-tone roots; it helps either to look at the chord note-by-note and work out each interval, or to transpose the chord up or down by int 0.5 in order to make it easier to identify. To assist the reader, I include a list of conventional triads and seventh chords with quarter-tone roots in Appendix B.

parts. The roots of neutral triads can be conventional pitches, such as F♮ or A♯, or quarter-tone pitches, such as G𝄲 (Example 1.16b). As is the case with all chords containing quarter-tone intervals, the neutral triad is always spelled with a combination of conventional and quarter-tone accidentals.[19]

Pitch Collections: Sets, Scales, and Interval Cycles

The conventions of pitch-class set notation provide a convenient means to create a taxonomy of chords. Following Allen Forte's method for identifying set-classes, I have written a simple computer program for generating a set list in equal-tempered universes ranging in size from 6 to 24 divisions of the octave. I use the letter 'c' (the first letter of the term 'cardinality') to represent the number of equal-tempered divisions in an octave. For example, c=12 represents conventional 12-note equal temperament, and c=24 represents the twenty-four equal divisions of the octave in the quarter-tone universe. My program, which assumes transpositional and inversional equivalence

[19] It is fairly easy to learn to recognize neutral triads that have conventional pitches as their roots, most likely because we are trained to recognize major and minor triads with conventional roots, and we can imagine the neutral triad as a sort of "average" between major and minor triads. It is more challenging to recognize neutral triads that have quarter-tone pitches as their roots, an ability that requires considerable practice. I provide a complete list of the twenty-four transpositions of the neutral triad in Appendix B.

among sets, generates as its output a simple text file containing a list of all unique sets in a given universe (excluding dyads and their complements), in normal order, along with their interval vectors.[20] The quarter-tone set list generated by my program for c=24 turned out to be impractically large; the program's output was a text file 20.9 megabytes in size that lists 352 671 unique quarter-tone sets ranging in size from trichords to 22-note sets (trichordal complements in c=24).[21] The sheer size of the quarter-tone set list makes set labels similar to Forte's impractical to work with. There are 256 quarter-tone tetrachords, a greater number of sets than the 208 conventional pitch-class sets that constitute the entire set list for c=12. Instead of set labels, I normally identify pitch-class sets with a string of pitch-class decimals separated by spaces and enclosed within curly braces; I list a set of intervals as a string of successive interval-classes enclosed within angle brackets as shown in Example 1.17.

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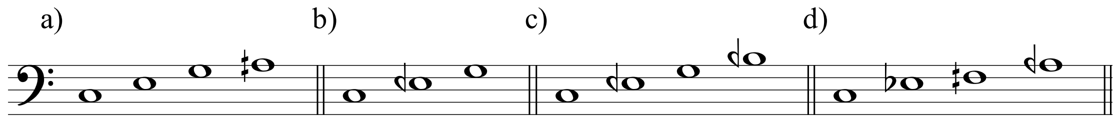
[20] The set list for c=6 is a trivially short list consisting of three self-complementary trichords, and the set list for c=7 corresponds to John Clough's set list for his diatonic set theory. The set list for c=12 contains all of the sets in Allen Forte's set list, but they are not listed in Forte's order, because the current version of my program does not take into account potential Z-relationships between sets with equivalent interval vectors.

[21] To print out the complete list would require 6298 standard single-spaced typewritten pages, or as much paper as the first seven volumes of *The New Grove Dictionary of Music and Musicians* plus the first 747 pages of the eighth volume. In making this comparison, I am counting only numbered pages. If you look at the first eight volumes of Grove sitting side-by-side on a bookshelf, you get a good idea of just how big the quarter-tone set list actually is.



{0.0 4.0 7.0 9.5} {0.0 3.5 7.0} {0.0 3.5 7.0 10.5} {0.0 3.0 5.5 8.5}

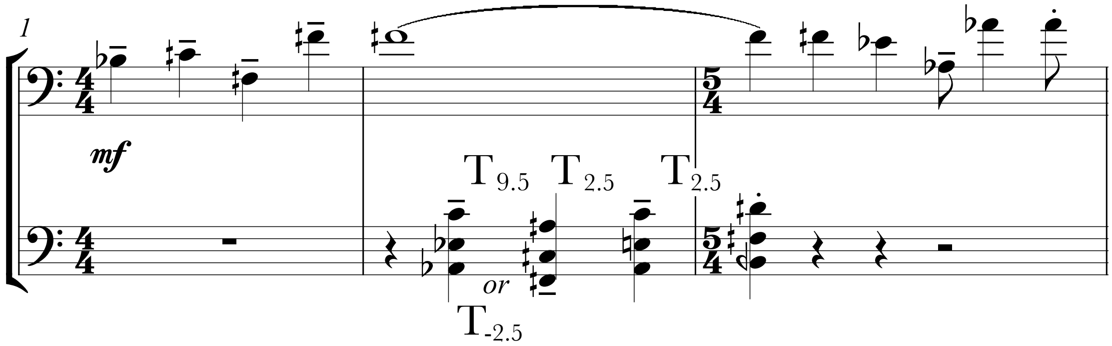
<4.0 3.0 2.5 2.5> <3.5 3.5 5.0> <3.5 3.5 3.5 1.5> <3.0 2.5 3.0 3.5>

Example 1.17: a) Hába's tetrachord; b) neutral triad; c) Ives's tetrachord; d) Wyschnegradsky's tetrachord

I have chosen four sets to illustrate my notational convention in Example 1.17. Below the staff, I label each set as a string of pitch classes on one line, and as a string of intervals on the second line.[22] The set in Example 1.17a, which is the final chord of Hába's *String Quartet No. 3,* is made up of a major triad {0.0 4.0 7.0} plus an additional pitch-class int 9.5 above the root of the triad. I can describe Example 1.17a as a set of pitch classes {0.0 4.0 7.0 9.5}, or as a succession of intervals <4.0 3.0 2.5 2.5>. In some cases, I may prefer to describe Example 1.17a as a literal set of pitches, in which case I would label the set as {C, E, G, A‡}. Example 1.17b shows a neutral triad {0.0 3.5 7.0} with a root of C. The pitch-class set notation makes it easier to recognize that the chord in Example 1.17c, a chord favoured by Ives, is a superset of the neutral triad with the addition of pc 10.5. However, Ives does not see this chord as a neutral triad with an added note, but rather as a

[22] Were we to use set labels, Hába's tetrachord would be set 4-243, the neutral triad would be 3-45, Ives's tetrachord would be 4-249, and Wyschnegradsky's tetrachord would be 4-254.

chord generated by a series of successive neutral thirds built above a root. In this case, the interval notation <3.5 3.5 3.5 1.5> makes it easy to recognize the cycle of int 3.5 that generates this chord. The chord in Example 1.15d, is a chord that Wyschnegradsky uses as a structural tonic in his *24 Preludes dans l'échelle chromatique diatonisée à 13 sons.* In fact, I find it more useful to think of this chord not as an abstract pitch-class set, but rather as a tonic chord made up of the first, fourth, seventh, and tenth steps of the scale shown in Example 1.20i below.

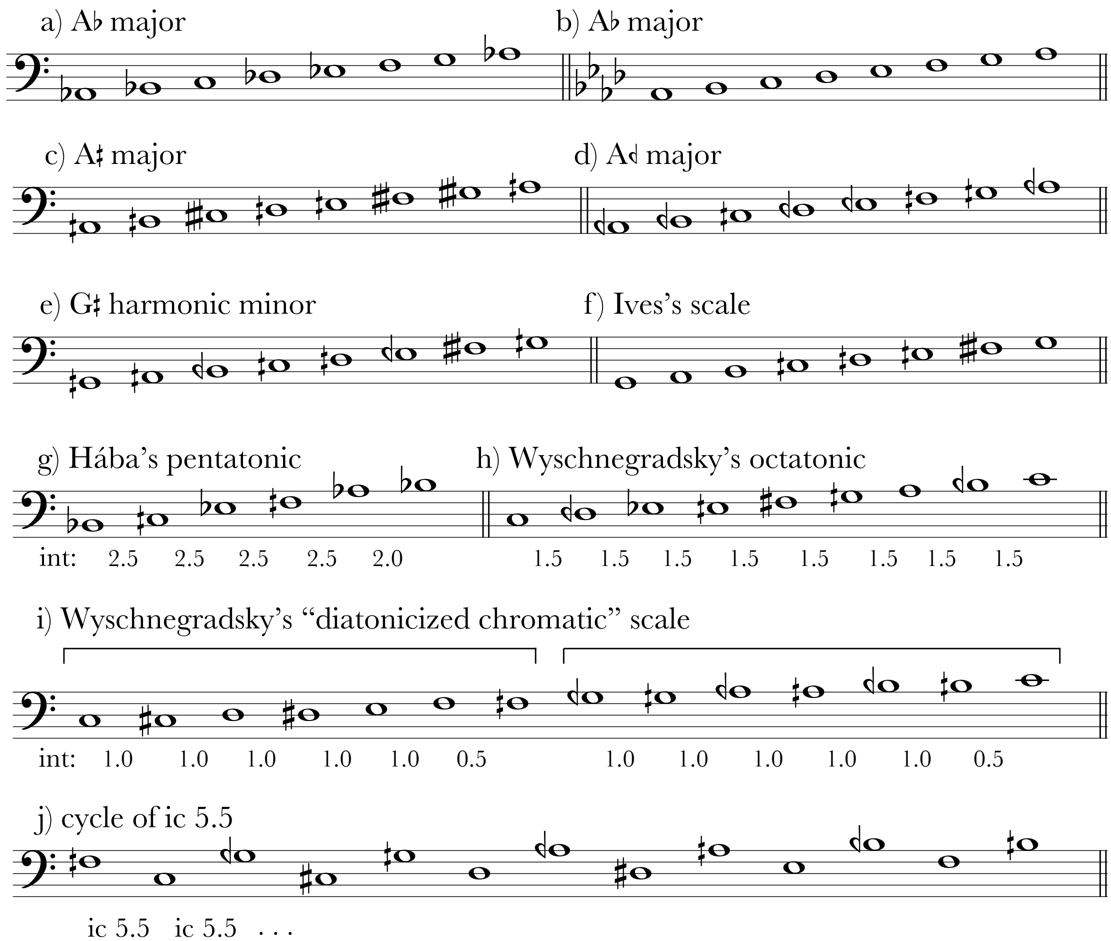


Example 1.18: Hába, *Suite für vier Posaunen im Vierteltonsystem,* Mvt. V, *Allegro risoluto,* mm. 1-3

At times, it is convenient to use T-operators to identify transpositions. The subscript attached to the T-operator is the interval of transposition. I have included a short excerpt from Hába's trombone quartet in Example 1.18. The three lower parts move in parallel major triads; the first triad is transposed down by int 2.5, followed by two successive upward transpositions by int 2.5. I indicate the transposition up by int 2.5 with the symbol "$T_{2.5}$" and the transposition down by int 2.5 either by "$T_{9.5}$" (which assumes octave equivalence), or by "$T_{-2.5}$" (the negative sign indicates a transposition downward).

Example 1.19: Interval cycles

The repeated application of a single T-operator generates an interval cycle. Conventional intervals generate familiar conventional interval cycles, only there are twice as many unique transpositions of each cycle in c=24 than there are in c=12. As I show in Example 1.19, there are now two unique transpositions of the chromatic scale (a $T_{1.0}$ cycle), four whole-tone scales ($T_{2.0}$ cycles), six diminished-seventh chords ($T_{3.0}$ cycles), eight augmented triads ($T_{4.0}$ cycles), two circles of fifths ($T_{5.0}$ cycles), and twelve tritones ($T_{6.0}$ cycles). All of the cycles generated by quarter-tone intervals (such as the cycle of neutral thirds in Example 1.19g) exhaust the full gamut of 24 pitches except for two: $T_{1.5}$ and $T_{4.5}$ both generate eight-note cycles that have only three unique transpositions (Examples 1.19h and Example 1.19i). Both cycles generate sets with identical pitch content; the first transposition of the $T_{1.5}$ cycle in Example 1.19h contains the same eight pitches as the first transposition of the $T_{4.5}$ cycle in Example 1.19i.



Example 1.20: Quarter-tone scales

The new quarter-tone pitches open up the possibility of new scales. The first scale in Example 1.20 is a conventional A♭–major scale, which can be written with accidentals placed before the individual pitches that require flats (Example 1.20a), or with a key signature of four flats with the flats placed in the traditional order of B♭, E♭, A♭, and D♭ (Example 1.20b). The scale in Example 1.20c is an A♭–major scale transposed up by three quarter tones, which results in a major scale with a quarter-tone tonic pitch, A𝄲. Because the conventional major scale contains nothing but conventional intervals, major scales with quarter-tone pitches as their tonics require a quarter-tone accidental for each pitch. A key signature for a major scale with a quarter-tone tonic will always require seven accidentals. The key signature for A𝄲 major would require three three-quarter sharps and four quarter-sharps, which could be placed in the traditional order for sharps: F♯, C♯, G♯, D𝄲, A𝄲, E𝄲, and B𝄲. It is difficult to decide on an appropriate ordering for the accidentals of the key signature for A𝄳 major (Example 1.20d) because the scale appears to mix flat and sharp accidentals, containing four pitches that require quarter flats {B𝄳, E𝄳, A𝄳, D𝄳} and three pitches that require quarter sharps {F𝄲, C𝄲, G𝄲}. It is also possible to construct conventional scales that require three different quarter-tone accidentals, such as the G𝄲 harmonic minor scale in Example 1.20e.

In general, quarter-tone composers have chosen to create new scales, although some composers will use quarter tones to modify conventional scales. In the third movement of his *Three Quarter-Tone Piano Pieces,* Ives distorts a G–major scale by transposing $\hat{4}$, $\hat{5}$, $\hat{6}$, and $\hat{7}$ up by one quarter tone to create the new scale in Example 1.20f. Hába uses the symmetrical pentatonic scale in Example 1.20g for the melody in the fifth movement of his trombone quartet. The quarter-tone pitches C𝄲 and F𝄲 divide the two perfect fourths B♭–E♭ and E♭–A♭ into two equal parts. This pentatonic scale is an example of a scale that can be generated by an interval cycle, in this case a cycle of int 2.5. Wyschnegradsky proposes an octatonic scale in which each scale step is the same size (Example 1.20h); this scale can be generated by int 1.5, an interval that divides the octave into eight equal parts. This scale, like the conventional octatonic scale, contains eight scale-steps and can be partitioned into two separate diminished seventh chords.[23] It also shares some of the properties of the conventional whole-tone scale because all scale-steps are the same size and there are a limited number of distinct intervals. Wyschnegradsky generates the scale in Example 1.20i by taking the pitches of the cycle of ic 5.5 in Example 1.20j and arranging them in scalar order. This scale can be divided into the two transpositionally equivalent heptachords that are bracketed in Example 1.20i. Wyschnegradsky calls this scale a "diatonicized chromatic" scale because it shares properties with both the diatonic major scale and the chromatic scale. The heptachordal structure of Wyschnegradsky's scale is analogous to the tetrachordal structure of the

[23] In Example 1.20g, the scale can be divided into the two fully diminished seventh chords {C, E♭, F♯, A} and {D𝄳, E𝄲, G𝄲, B𝄳}. Every transposition of Wyschnegradsky's scale can be partitioned into one diminished seventh chord made up of conventional pitches, and one made up of quarter-tone pitches.

diatonic major scale, while its pitch density and large number of semitonal scale-steps recall the conventional chromatic scale.

Conventions of Analytical Sketch Notation

Example 1.21: Hába, *String Quartet No. 3,* Mvt. III, mm. 7-14

Throughout this dissertation, I will illustrate my observations with analytical sketches, using a notation that bears a superficial resemblance to Schenkerian sketch notation. The particular variant of sketch notation that I use is heavily influenced by the sketch style of Charles Smith.[24] In a typical sketch, the notation differentiates chord tones that belong to some referential sonority from non-chord tones, which are subordinate notes that embellish or prolong the chord tones. In tonal music, the referential sonorities are traditional triads and seventh chords and the embellishments include familiar dissonances such as passing tones, neighbour notes, suspensions, and appoggiaturas. In my sketches, the concept of "chord tone" varies from composer to composer. While some composers (such as Blackwood and Hába) privilege traditional triads and seventh chords as their referential sonorities, other composers (such as Ives and Wyschnegradsky) invent new, unfamiliar chords to serve as referential sonorities.

To help illustrate the sketch notation, I have sketched a short excerpt from Hába's *String Quartet No. 3* (Example 1.21). I have included mm. 7-14 from the actual score in the top system, vertically aligned with two levels of sketch notation so that the pitches of the sketches line up directly beneath the

[24] Charles J. Smith, *Tonal Models of Music,* unpublished MS, Introduction to Vol. 2, pp. 7-13

musical events that they represent. The second system in Example 1.21 is a local-level or foreground sketch, and the third system is a larger-scale middleground sketch.[25] The barlines in the sketch show where one chord ends and the next begins; these barlines indicate changes of harmony and do not necessarily coincide with the actual metrical barlines in the music. I use the white noteheads to represent chord tones; in this example, I am assuming that chord tones are members of triads. The five measures of the foreground sketch represent the five chords in this excerpt: (1) a triad with a root of C (an incomplete triad missing its third) spans mm. 7-8; (2) a {G, B‡, D} triad spans mm. 9-10; (3) a complete C–major triad extends from m. 11 through beat 2 of m. 13; (4) a D‡–major triad {D‡, F♯, A‡} spans the last eighth-note of m. 13 and the first quarter note of m. 14; and (5) the passage ends with a C–major triad on the second quarter-note of m. 14. The black noteheads represent non-chord tones; for example, the upper-neighbour pitches D‡ and

[25] In general, my quarter-tone sketches do not include deep middleground or background levels, because I have no evidence that there is anything analogous to an *Ursatz* underlying the structure of any of the quarter-tone music I have studied. At this point, it is premature to speculate about background structures in quarter-tone music. Even middleground sketches nearer the musical surface present problems, because there are no established criteria for deciding which chords are structurally significant and which chords serve to embellish them. In Example 1.21, I show the D‡–major triad as an embellishment of the C–major triad because the progression C–D‡–C resembles the plagal progression I–V/V–I in C major. I show the soprano line G–A‡–C as a passing line even though A‡–C is not spelled as a stepwise interval. We can hear int 2.5 as a large step (or large major second), even though when it divides a perfect fourth into two equal parts, one part will be spelled as a second, and the other part will be spelled as a third (see Chapter 4).

A𝄲 in m. 8 embellish the pitches C and G of the first chord. The stemmed notes of the soprano and bass indicate which pitches form the underlying outer-voice counterpoint. The numerals placed between the staves identify the generic intervals between the outer voices.

The sketch notation highlights some interesting features of this passage. The first three chords show a root succession of C, G, and C with a typical 5-8-5 outer-voice contrapuntal configuration, a pattern that resembles I-V-I in C major, even though the first C–triad is missing its third, and the G–chord has an altered third, B𝄲 instead of B♮. However, a purely tonal interpretation of this passage is difficult to justify in light of what follows the apparent I-V-I. Conventional Roman numerals cannot situate the D𝄲–major triad within the context of C major, although the root of D𝄲 suggests some sort of chromatically altered ii chord with quarter-tone pitches, and the third of the chord, F♯ ($\sharp\hat{4}$ in C major) suggests an altered secondary dominant of V. In the middleground level of the sketch, I have interpreted the D𝄲–major chord as a neighbour chord prolonging the C–major chord of mm. 11-14. The passing-tone A𝄲 in the soprano performs a function that is not possible with conventional tuning; the A𝄲 divides the perfect fourth G-C (int 5.0) into two equal parts.

The middleground level of the sketch gives the impression that this passage displays little more than a conventional tonal middleground with a few quarter-tone accidentals thrown in for colour. The phrase is in C major: as I have sketched it, the harmonic progression appears to be a straightforward I–V–I with all three triads in root position, and the second tonic prolonged by a quarter-tone neighbour chord. It is tempting to conclude that because we can find tonal patterns in Hába's music, that Hába's music must therefore be tonal. This conclusion is based on a logical fallacy—it is true that Hába's music will mimic tonal idioms, but that fact alone does not make it tonal. He does not typically follow the norms of common-practice tonality such as harmonic syntax, voice-leading, or outer-voice counterpoint. The motion from the C–major triad to the D‡–major triad involves parallel fifths in the outer voices, which in traditional tonal styles would ordinarily be considered poor counterpoint.[26]

Throughout this dissertation, I examine how composers borrow common-practice tonal conventions without actually writing tonal music. As stated

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[26] In order to eliminate the parallel fifths in the outer voice counterpoint, we might be tempted to consider the F♯ as the bass of a D‡ chord in $^6_3$ inversion. However, if this passage were tonal, F♯ (♯$\hat{4}$) would most likely function as a leading tone to G. This function is thwarted because the bass pitch which follows the F♯ is not the expected G, but rather C. In fact, outer-voice parallel-fifth voice-leading configurations such as the one in Example 1.21 appear multiple times throughout the piece, and so do not represent contrapuntal errors, but rather an important motive.

above, Chapters 2 through 5 present analyses of selected quarter-tone works by Easley Blackwood, Alois Hába, Charles Ives, and Ivan Wyschnegradsky, beginning with the most conservative and ending with the most progressive. In Chapter 2, I examine Blackwood's single quarter-tone composition, *24 notes: Moderato.* Blackwood's quarter-tone writing extends the strict 16th-century contrapuntal models of dissonance preparation and resolution to incorporate new quarter-tone intervals. The harmonic vocabulary in *24 notes* is conservative, consisting of conventional triads and seventh chords with quarter-tone embellishments. In Chapter 3, I examine Hába's trombone quartet, *Suite für vier Pausonen im Vierteltonsystem, Op. 72.* Hába, as we have already seen, uses triads and root motion by perfect fifth to invoke tonal conventions. While quarter tones function primarily as embellishments of conventional triads, but there is one significant chord that includes a quarter-tone added sixth (Example 1.17a). In Chapter 4, I examine Ives's *Three Quarter-Tone Pieces.* Ives's quarter-tone music also makes reference to conventional tonal idioms, but unlike Hába, Ives's referential sonorities are not conventional triads, but cyclically-generated quarter-tone sets such as the one shown in Example 1.17c. In *Three Quarter-Tone Pieces,* the quarter tones do not function as mere embellishments, but rather form an important component of structurally significant harmonies. In Chapter 5, I examine

Wyschnegradsky's *24 Preludes dans l'echelle chromatique diatonisée à 13 sons, Op. 22.* Wyschnegradsky generates pitch content in these preludes through a technique that he names "diatonicized chromaticism," whereby he creates a new quarter-tone scale with properties similar to the conventional major scale. From this scale, Wyschnegradsky derives a quarter-tone chord that functions as a conventional tonic. In Wyschnegradsky's music, quarter tones do not function merely as embellishments of conventional pitches; instead, they are integrated into structurally significant harmonies such as a tonic chord. No evidence suggests that Wyschnegradsky seeks to "reinvent" tonality; however, I have found specific configurations in *24 Preludes* that mimic typical prolongations of tonic harmony. Chapter 6 reinforces the relationship between Wyschnegradsky's scale and the conventional major scale, considering the intersection between Wyschnegradsky's diatonicized chromaticism and neo-Riemannian transformational theory. By applying Cohn's definition of a generalized trichord to the diatonicized chromatic scale, I derive quarter-tone equivalents to the canonic neo-Riemannian transformations P, L, and R that operate on Wyschnegradsky's quarter-tone tonic chord.[27]

[27] Cohn, "Neo-Riemannian Operations, Parsimonious Trichords, and Their *Tonnetz* Representations."